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Original Research Paper

Application of Fractional Factorial Design And Dimensional Analysis for Analysis of Main Cutting Force in Turning

Primena frakcionog faktornog plana i dimenzionalne analize za analizu glavnog otpora rezanja kod struganja

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Abstract: *Cutting forces are very important aspect of machining processes because they directly affect heat generation and cutting power, tool life and energy consumption. Cutting forces can also be used as an auxiliary criterion for the evaluation of material machinability. This paper presents the application of dimensional analysis (DA) model for prediction of the main cutting force in dry longitudinal single-pass turning of not heat-treated non-alloyed and low-alloyed steels with carbon content higher than 0.55%. A fractional factorial design was used to arrange six parameters: depth of cut, feed rate, cutting speed, rake angle, cutting edge angle and tensile strength as workpiece material mechanical property. The machining calculator of the cutting tool manufacturer was used for estimating the main cutting force for five workpiece material groups. The analysis of the results encompassed the evaluation of main and interaction effects of the considered parameters, the development of quasi-linear and DA-based prediction models, and the interpretation of the DA model coefficients. The experimental and validation results indicated that the DA model has good potential for prediction of the main cutting force in turning processes.*

Keywords: main cutting force, dimensional analysis, turning, modeling

Apstrakt: *Otpori rezanja su veoma važan aspekt procesa obrade jer direktno utiču na stvaranje toplote, snagu rezanja, postojanost reznog alata i potrošnju energije. Otpori rezanja se mogu koristiti kao pomoćni kriterijum za procenu obradljivosti materijala. Ovaj rad predstavlja primenu modela dimenzionalne analize (DA) za predikciju glavnog otpora rezanja kod suvog uzdužnog jednoprolaznog struganja nelegiranih i niskolegiranih čelika sa sadržajem ugljenika većim od 0,55%. Primenjen je frakcioni faktorni plan za sagledavanje šest parametara: dubina rezanja, korak, brzina rezanja, grudni ugao, glavni napadni ugao i zatezna čvrstoća kao mehaničko svojstvo materijala. Kalkulator obrade proizvođača reznog alata je korišćen za procenu glavnog otpora rezanja za pet grupa materijala obratka. Analiza rezultata je obuhvatila procenu glavnih i uticaja interkacija razmatranih parametara, kreiranje kvazilinearnog i DA modela, kao i interpretaciju koeficijenata DA modela. Eksperimentalni i validacioni rezultati su pokazali da DA model ima dobar potencijal za predikciju glavnog otpora rezanja kod struganja.*

Ključne riječi: bušenje, habanje, sila, hrapavost

1 INTRODUCTION

Measuring cutting forces is among the most effective techniques for detecting cutting conditions. During the turning process, these forces directly influence heat generation, which in turn impacts the dimensional accuracy of the machined part, surface quality, and tool wear [1]. Knowing the cutting force estimates in advance is also essential for fixture design [2]. Additionally, cutting forces are utilized to estimate cutting power and the energy consumption during the machining process, which are critical considerations in modern manufacturing [3]. Knowing the cutting forces for different cutting regimes helps the designer-manufacturer to increase the efficiency of machine tools. Cutting forces depend on tool geometry, workpiece material, feed rate, depth of cut, cutting speed, cutting insert coatings, cutting fluid, etc. [4].

The rapid advancement of technologies, novel cutting tool coatings and workpiece materials necessitates precise understanding of cutting forces, making their investigation during turning a critical area of research [5]. In practice, cutting forces are usually estimated using a number of different empirical models [2].

Different researchers have used different approaches to predict the cutting forces, including analytical, numerical, experimental methods, machine learning (ML) and hybrid models. Tang et al. [6] developed exponential and quadratic polynomial empirical models for predicting three-component cutting forces using five factors (cutting speed, depth of cut, feed rate, workpiece material hardness and tool nose radius) in dry hard turning of AISI D2 steel with the PCBN cutting tool. Models were developed using orthogonal regression methodology (ORM) and response surface methodology (RSM). The experimental results demonstrated that the RSM quadratic polynomial empirical model exhibits significantly greater accuracy and reliability compared to the ORM exponential empirical model. Parihar et al. [7] developed a numerical model to predict cutting forces based on cutting speed, feed rate and depth of cut when turning AISI H13 steel using a ceramic cutting tool. DEFORM 3D software, utilizing the finite element method (FEM) with a Lagrangian formulation, was employed to predict cutting forces. The simulation and experimental data were compared, demonstrating good agreement. Alajmi and Almeshal [8] modeled the cutting force in turning of AISI 4340 alloy steel by using Gaussian process regression (GPR), support vector machines (SVM) and artificial neural network (ANN) ML models. Performance comparisons of these models indicated that GPR is an effective approach, capable of delivering high predictive accuracy for cutting force estimation.

This study deals with the analysis of the effects of cutting parameters on the main cutting force in dry longitudinal single-pass turning of not heat-treated non-alloyed and low-alloyed steels with carbon content higher than 0.55% using data from Walter machining calculator [9]. Walter workpiece material groups P2, P3, P4, P7 and P14 were considered. In the present study fractional factorial design 2^{5-1} was applied to assess the effects of six parameters, three related to the machining process (depth of cut, feed rate, and cutting speed), two related to the geometry of the cutting tool (rake angle and cutting edge angle), and one parameter of the workpiece material (tensile strength) [10]. In addition, a DA model was developed for the prediction of the main cutting force. The analysis of the obtained results included the analysis of the effects of considered parameters on the main cutting force and the analysis of DA model’s coefficients.

2 DIMENSIONAL ANALYSIS

Dimensional analysis (DA) is a technique used to identify and establish relationships between physical quantities by analyzing and comparing their dimensions [11]. DA involves examining the relationships between various physical quantities to identify their fundamental dimensions: mass (M), length (L), time (T), temperature (θ) and current (I) [12]. Also, it deals with the way to transform the measurement units that are related to those quantities by moving from one physical quantity to another [13]. This results in each group member having the same dimensional representation, which is the law of dimensional homogeneity [14].

Buckingham's π-theorem in dimensional analysis establishes the number of dimensionless groups required to characterize the relationships among variables, enabling a complete description of the process under investigation [15]. Applying the π-theorem reduces the number of variables in the phenomenon being studied, thereby reducing the required number of measurements and facilitating the analysis of the results. The theorem is generally realized through the following steps [15].

- Physical quantities which are assumed to govern the physical phenomenon are considered. A table is compiled listing their symbols and dimensions or measurement units, from which the number of dimensionally independent quantities is determined.
- From the set of physical quantities, select the dimensionally independent quantities and demonstrate their dimensional independence according to the given theorem.
- Dimensionless π parameter is formed from every physical quantity outside the set of dimensionally independent quantities in such a way that its dimension is represented by dimensions of physical quantities from a dimensionally independent set.

3 EXPERIMENTAL DATA AND DA MODEL

The cutting tools were tool holders PCLNR2525M12 (cutting edge angle of κ = 95°, rake angle of γ_{oh} = -6°) and PCBNR2525M12 (κ = 75°, γ_{oh} = -6°), with CNMG120412-MP3 WPP05S (γ_{oi} = 22.5°) and CNMG120412-MP5 WPP05S (γ_{oi} = 15°) cutting inserts. The turning diameter was set to 72 mm, length of cut to 50 mm, and machine tool efficiency to 0.8 [10].

Parameters and their values at low (-1) and high (+1) level are shown in Table 1. Parameter ranges and levels are selected based on the capability of the Walter machining calculator and the recommended cutting conditions of the cutting tools manufacturer. Main cutting force data acquisition was performed using Walter machining calculator by applying standard fractional factorial design 2^{5-1}.

Values of tensile strength for different workpiece material groups are shown in *Table 2*.

Table 1 – Parameters with their names, units and labels

Parameter	Unit	Low level (-1)	Low level (+1)	Label
Depth of cut, a_p	mm	1.2	3.5	A
Feed rate, f	mm/rev	0.2	0.4	B
Cutting speed, v	m/min	280	313	C
Rake angle, γ_0	°	9	16	D
Cutting edge angle, κ	°	75	90	E

Table 2 – Tensile strength for different workpiece material

Parameter	Unit	Walter workpiece material group				
		P2	P3	P4	P7	P14
Tensile strength, R_m	N/mm^2	639	708	639	591	675

Based on the fractional factorial design, 16 different combinations of parameter levels were tested in a "virtual" experiment and the values of the main cutting force (F_c) were obtained upon which one can estimate the main and interaction effects of the selected parameters. It has to be noted that machining calculator uses Kienzle's cutting force model with the unit specific cutting force value of $k_{c1.1}$ = 1700 N/mm^2 and exponent value of m_c = 0.25 for multiple workpiece material groups (P2, P3, P4, P7 and P14).

For the development of DA model for the estimation of the main cutting force the following procedure was adopted. First, the dimensions and parameters affecting the main cutting force are determined as follows [16]:

- Main cutting force (F_c): MLT^{-2}
- Tensile strength (R_m): $ML^{-1}T^{-2}$
- Depth of cut (a_p): L
- Feed rate (f): L
- Cutting speed (v): LT^{-1}
- Feed velocity (v_f): LT^{-1}

The dependent parameter is the main cutting force, and the independent parameters are the tensile strength (P7 workpiece material group), depth of cut, feed velocity, rake angle and cutting edge angle. Feed rate, cutting speed and rake angle are selected as repeating parameters. The number of dimensions is 3 (L, T, and M). In this case there are four dimensional groups, π_1, π_2, π_3, π_4 represented by the equations (1) – (4):

$$\pi_1 = \frac{F_C}{R_m \cdot f^2} \tag{1}$$

$$\pi_2 = \frac{v_f}{v} \tag{2}$$

$$\pi_3 = \frac{a_p}{f} \tag{3}$$

$$\pi_4 = \frac{\kappa}{\gamma_0} \tag{4}$$

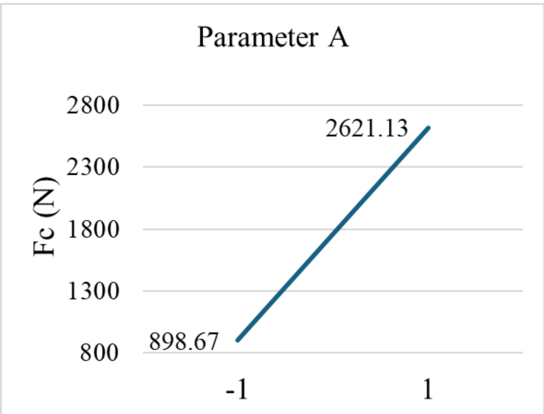
If one set: $\pi_1 = f(\pi_2, \pi_3, \pi_4)$, the main cutting force as the function of dimensional groups can be represented by the following equation:

$$F_c = x_1 \cdot f^2 \cdot R_m \cdot \left(\frac{v_f}{v}\right)^{x_2} \cdot \left(\frac{a_p}{f}\right)^{x_3} \cdot \left(\frac{\kappa}{\gamma_0}\right)^{x4} \tag{5}$$

The unknown coefficients of the DA model were determined based on the main cutting force data by minimizing the sum of squared error. They were determined as: x_1=0.658385, x_2=-0.250038, x_3=0.999975 and x_4=0.109698. The mean absolute percentage error (MAPE) of the DA model is 1.73 %.

4 RESULTS AND DISCUSSION

To estimate the effect of each model, the difference between the mean response values at the high (+) and low (−) levels is calculated [17]. The individual influences of the considered parameters on the main cutting force are illustrated in *Figure 1*.



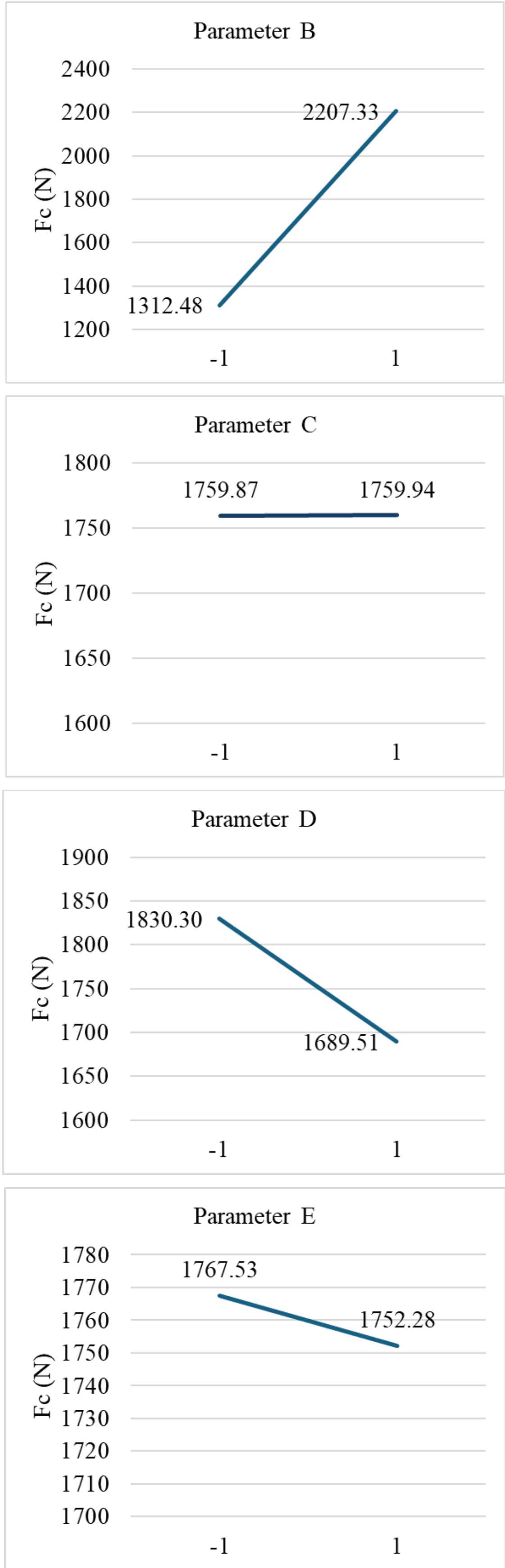


Figure 1 – Main effects plots of considered parameters on the main cutting force for low-alloyed steels P7

Parameters A and B, i.e., depth of cut and feed rate have a positive correlation with the main cutting force, i.e., by increasing the depth of cut or feed rate, the main cutting force increases sharply. Clearly, an increase in the un-deformed chip cross section and the volume of the deformed material, either by increase in depth of cut or feed rate, will result in an increase in the main cutting force.

For parameters C and E, i.e., cutting speed and cutting edge angle, it can be said that the results show a negligible, imperceptible influence on the main cutting force. Indeed, as discussed by Stephenson and Agapiou [2] for most materials (except soft metals and temperature-sensitive materials) cutting speed has a minor effect on the main cutting force for a wide range of cutting conditions.

On the other hand, parameter D, i.e., rake angle, has a negative correlation. An increase in the rake angle leads to a reduction in the main cutting force. This occurs because a larger rake angle increases the shear angle, which decreases chip thickness and consequently lowers both the main cutting force and the required cutting power [18, 19], which is consistent with the observation from Figure 1. Although high rake angles decrease the main cutting force, they at the same time reduce the strength of the tool tip which ultimately may lead to its fracture [20].

For the analysis of statistical significance of five main parameter effects and ten two-way interactions, Lenth's pseudo standard error (PSE) approach [21] was applied. In the present study, for n=15 considered effects and corresponding v=3 degrees of freedom in the t distribution, the estimated PSE is 2.17. Therefore, all effects whose absolute values are larger than 5.57 (margin of error) should be considered significant (*Figure 2*).

As could be observed from Figure 2 main effects of depth of cut (A), feed rate (B), rake angle (D) and cutting edge angle (E) are statistically significant along with interaction effects of the depth of cut and feed rate (AB), depth of cut and rake angle (AD), feed rate and rake angle (BD) and cutting speed and cutting edge angle (CE).

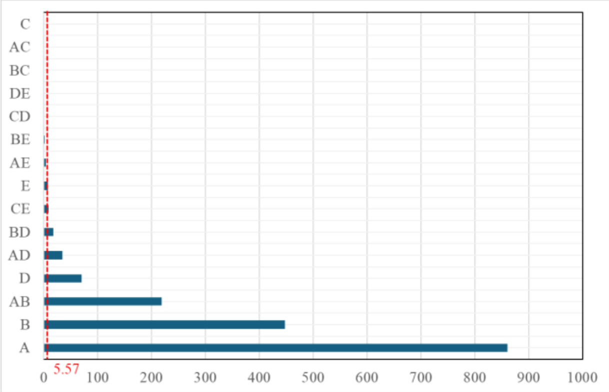


Figure 2 – Pareto chart of the main and interaction effects

However, although these interactions are significant, there is no qualitative change in the influence of parameters on the main cutting force, i.e., their influence is more or less pronounced depending on the setting of other parameters. For example, the importance of interaction of depth of cut and feed rate is reflected in the fact that when depth of cut is set at higher level, an increase in feed rate results in a greater increase in the main cutting force than in the case when depth of cut is set at low level. Similarly, increasing the rake angle when depth of cut is at high level decreases the main cutting force somewhat greater.

Based on conducted analysis one can derive the quasi-linear model for the prediction of the main cutting force which includes only statistically significant terms in the following form:

$$F_c = 1759.9 + 861.2 \cdot a_p + 447.4 \cdot f - 70.4 \cdot \gamma_0 - 7.63 \cdot \kappa + 219 \cdot a_p \cdot f - 34.5 \cdot a_p \cdot \gamma_0 - 17.9 \cdot f \cdot \gamma_0 - 8.76 \cdot v \cdot \kappa \quad (6)$$

For the entire experiment MAPE of the quasi-linear model is only 0.29% showing very good prediction capability.

The analysis of DA model and constituent π groups (equation 5) leads to the following conclusion. Namely, by increasing the π_1, π_3, π_4 groups, the main cutting force increases, the only opposite exists in the case of the π_2 group, where increasing the value decreases the value of the main cutting force.

In the same way, by using the DA model, the main cutting forces for the other Walter material groups (with different tensile strength values) can be estimated. The DA model coefficients were determined and shown in *Table 3*.

Table 3 – DA models' coefficients for different workpiece material groups

Walter workpiece material groups	DA model coefficients			
	x_1	x_2	x_3	x_4
Model 1 P2 (P4)	0.6089	-0.25003	0.999977	0.10969
Model 2 P3	0.5495	-0.25004	0.999972	0.10969
Model 3 P7	0.6583	-0.25003	0.999975	0.10969
Model 4 P14	0.5764	-0.25004	0.999975	0.10969
Model 5 Average	0.5983	-0.25003	0.999974	0.10969

The mean values of the coefficients were calculated to determine the model of the main cutting force. As could be observed from *Table 3*, only value of coefficient x_1 is changing as a corrective parameter for different values of workpiece material tensile strength.

The MAPE of the Model 3 from Table 3 is 1.73% while the MAPE of the main cutting force model with average coefficient values is 2.04. Although errors are slightly higher compared to the quasi-linear model, one may argue that the developed DA model proved to be quite accurate for the estimation of main cutting forces, not only for one workpiece material group, but also for multiple workpiece material groups. In order to check the validity of the developed main cutting force prediction models, the additional cutting regimes were tested (*Table 4*).

Table 4 – Estimated main cutting force values for different cutting regimes (parameters related to machining process and geometry of the cutting tool for P7 workpiece material group)

A a_p **(mm)**	**B** f **(mm/rev)**	**C** v **(m/min)**	**D** γ_0 **(°)**	**E** κ **(°)**	**Fc (N)**			
					Machining calculator	Model 3	Model 5	Quasi-linear model
2	0.3	300	9	80	1573.74	1555.014	1413.19	1560.86
3	0.4	280	12	90	2821.68	2840.727	2581.641	2834.39
3.5	0.25	310	14	75	2281.1	2245.212	2040.44	2261.64
1.75	0.3	290	10	80	1361.89	1345.006	1222.336	1349.40
2.5	0.35	305	16	75	2048.3	2034.068	1848.554	2038.53
1.5	0.1	350	9	90	515.81	518.293	471.0223	525.28
2.5	0.35	400	16	80	2038.41	2048.52	1861.688	2047.24
5	0.8	500	16	95	7578.58	7760.89	7053.068	8072.28
2	0.3	300	5	60	1696.57	1607.059	1460.487	1654.62
1.5	0.4	320	18	70	1335.25	1321.652	1201.114	1343.70

The obtained results show that the main cutting force values predicted by DA Model 3 are very close to the values obtained using Walter machining calculator. For all validation trials, the MAPE is only 1.51 %, which shows that the dimensional analysis provides means to develop quite accurate models for predicting the main cutting force in turning process. Considering average values of the coefficients for obtaining the model of the main cutting force (Model 5), the MAPE is 9.75 %, but still may be acceptable from manufacturing engineering point of view. On the other hand, the derived quasi-linear model showed approximately the same prediction deviations as Model 3 with MAPE of 1.54%, but somewhat greater when one considers results for the first 16 design points from the used fractional factorial design.

As could be observed from Table 4, the first five cutting regimes are within covered experimental hyper-space, while the others are outside experimental hyper-space, in terms of selected parameter values. Taking into account the obtained results one can argue that DA model provides good extrapolation result. However, as discussed by Stephenson and Agapiou [2], the use of empirical force models for extrapolating purposes should be ideally avoided but may be necessary, in some cases, to reduce testing requirements.

5 CONCLUSIONS

This study focused on analyzing effects of six parameters on the main cutting force in dry longitudinal single-pass turning of not heat-treated non-alloyed and low-alloyed steels with carbon content higher than 0.55%. By considering the main cutting parameters, the geometry of the tool and the parameter of the workpiece material, a fractional factorial design, and using Walter machining calculator, the main cutting force was estimated for different cutting regimes. By the application of the Lenth's method statistically significant main and interaction effects were determined. Moreover, based on acquired data and performed analyses quasi-linear and DA based models were developed for prediction of main cutting force for arbitrarily chosen sets of cutting parameters.

The results indicated that the DA cutting force prediction model provides approximate results to those obtained using machining calculator, with an approximate error of less than 1.73%, for P7 workpiece material group. By considering the average coefficients of all workpiece material groups, the model of the main cutting force gives approximate values with an average percentage error of slightly more than 2.04%.

It can be concluded that DA models show very good potential for estimation of the main cutting force in turning. Moreover, additional validation trials with parameter values outside initial experimental hyper-space, showed very good extrapolating capabilities of the DA model with low MAPE values. In comparison to classical quasi-linear model, it was observed that DA model can provide similar or even slightly better results, when it comes to extrapolation, and that with a much smaller number of terms in the model to be estimated.

Given the nature of the DA model and the number of unknown coefficients to be estimated, one can argue that application of DA offers significant benefits regarding the number of needed experimental trials, time and costs of experimentation, as well as possibility to consider multiple important parameters related to the cutting process, cutting tool geometry or workpiece material properties.

Based on developed DA model and estimated coefficients, one can easily determine the effects of independent variables, but also, considering very good accuracy, one can conclude that DA based models can be used for main cutting force prediction in situations when multiple statistically significant interaction effects occur, but where there is no qualitative change in the parameter effects.

Experimental measurements and development of DA models for prediction of main cutting forces for different workpiece materials using low resolution designs with limited number of trials will be in the focus for the future research.

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